

$x+10$ (") $x -$.
 (") B - E - $y -$.
 :

s - "	v - "	t -	
$70-y$	x	$\frac{70-y}{x}$	E A -
y	$x+10$	$\frac{y}{x+10}$	B E -
y	x	$\frac{y}{x}$	E B -
$70-y$	$x+10$	$\frac{70-y}{x+10}$	A E -

$\frac{70-y}{x} + \frac{y}{x+10} = 4.5$, $4.5 - B - A -$

$\frac{y}{x} + \frac{70-y}{x+10} = 6$, $6 - A - B -$

$$\begin{cases} \frac{70-y}{x} + \frac{y}{x+10} = 4.5 \\ \frac{y}{x} + \frac{70-y}{x+10} = 6 \end{cases}$$

$$\begin{cases} (70-y)(x+10) + xy = 4.5x(x+10) \\ y(x+10) + x(70-y) = 6x(x+10) \end{cases}$$

$$\begin{cases} 70x + 700 - xy - 10y + xy = 4.5x^2 + 45x \\ xy + 10y + 70x - xy = 6x^2 + 60x \end{cases}$$

$$+ \begin{cases} 70x - 10y + 700 = 4.5x^2 + 45x \\ 70x + 10y = 6x^2 + 60x \end{cases}$$

$140x + 700 = 10.5x^2 + 105x$

$10.5x^2 - 35x - 700 = 0 \rightarrow x_{1,2} = \frac{35 \pm 175}{21} \rightarrow x = 10 \leftarrow x > 0$

" 10 :

$\frac{70-y}{10} + \frac{y}{10+10} = 4.5$: $x = 10$.

$140 - 2y + y = 90 \rightarrow y = 50$

" 50 B - E - :

"

$$a_k - a_n = (1) \cdot d$$

$$a_1 = md$$

$$a_n + a_k = a_1 + d(n-1) + a_1 + d(k-1)$$

$$a_n + a_k = a_1 + d(n-1) + md + d(k-1)$$

$$a_n + a_k = a_1 + d(n+k+m-2)$$

$$a_n = a_1 + d(n-1) \tag{2}$$

$$n+k+m-1 = a_n + a_k$$

$$n+k+m-1 =$$

$$n = 34, k = 65 \tag{1} \quad , a_{34} + a_{65} \tag{1}$$

$$a_{34} + a_{65} = a_1 + d(34+65+m-2)$$

$$a_{34} + a_{65} = a_1 + d(97+m)$$

$$a_1 + d(97+m) =$$

$$\tag{2} \quad , a_1 + d(97+m) = a_{109} \quad , a_{34} + a_{65} = a_{109} \tag{2}$$

$$109 = 34 + 65 + m - 1$$

$$\boxed{m = 11}$$

$$7900$$

$$79$$

$$7900 = \frac{79}{2} (2a_1 + 78d)$$

$$100 = a_1 + 39d$$

$$100 = md + 39d$$

$$100 = 11d + 39d$$

$$\boxed{d = 2}$$

$$a_1 = md = 11 \cdot 2$$

$$\boxed{a_1 = 22}$$

$$a_1 = 22, d = 2 :$$

3 2 -

(1) .

:

$$k = 2, p = \frac{1}{6}, n = 3, \quad ,$$

$$P_3(2) = \binom{3}{2} \left(\frac{1}{6}\right)^2 \left(1 - \frac{1}{6}\right)^{3-2} = 3 \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^1 = \frac{5}{72}$$

$$\cdot \frac{5}{72} \quad :$$

: -

3 2 -

(2)

$$k = 2, p = \frac{1}{3}, n = 3, \quad ,$$

$$P_3(2) = \binom{3}{2} \left(\frac{1}{3}\right)^2 \left(1 - \frac{1}{3}\right)^{3-2} = 3 \cdot \left(\frac{1}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^1 = \frac{2}{9}$$

$$\cdot \frac{2}{9} \quad :$$

:

0.5 ,

(1) .

$$p = 0.5 \cdot \frac{5}{72} + 0.5 \cdot \frac{2}{9} = \frac{7}{48}$$

$$\cdot \frac{7}{48} \quad :$$

: (2)

$$\begin{aligned} & P(\text{Unbalanced Cube} / 2 \text{ six out of } 3) = \\ & = \frac{P(\text{Unbalanced Cube} \cap 2 \text{ six out of } 3)}{P(2 \text{ six out of } 3)} = \end{aligned}$$

$$= \frac{0.5 \cdot \frac{2}{9}}{\frac{7}{48}} = \frac{16}{21}$$

$$\cdot \frac{16}{21} \quad :$$

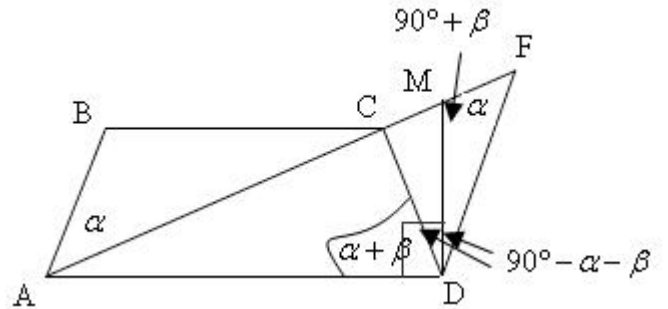
$$\cdot \left(\frac{2}{3}\right)^n \quad " \quad n$$

$$\cdot 1 - \left(\frac{2}{3}\right)^n \quad " \quad n$$

$$\cdot 1 - \left(\frac{2}{3}\right)^n \quad :$$

"

$\angle MAD = 90^\circ$.3 $AB \parallel DF$. 2 ($BC \parallel AD$) ABCD .1
 $\frac{AC}{AF} = \frac{MC}{MF}$. $\angle CDM = \angle MDF$. $\triangle ABC \sim \triangle FDA$. : "



	ABCD	4	1
	$BC \parallel AD$	5	1
	() $\angle BCA = \angle CAD = s$	6	5
	$AB \parallel DF$	7	2
	() $\angle AFD = \angle BAC = r$	8	7
	$\triangle ABC \sim \triangle FDA$	9	8,6
. . .			
	$\angle BAD = r + s$	10	8,6
+ "	$\angle CDA = \angle BAD = r + s$	11	10,4
	$\angle MAD = 90^\circ$	12	3
	$\angle CDM = 90^\circ - r - s$	13	12,11
$\triangle MAD$ -	$\angle FMD = 90^\circ + s$	14	12,6
180° - $\triangle FMD$ -	$\angle MDF = 90^\circ - r - s$	15	14,8
	$\angle CDM = \angle MDF$	16	15,13
. . .			

ΔFCD	$\frac{MC}{MF} = \frac{CD}{FD}$	17	16
	$AB = CD$	18	4
	$\frac{AB}{FD} = \frac{BC}{DA} = \frac{AC}{FA}$	19	9
	$\frac{AC}{AF} = \frac{MC}{MF}$	20	19,18,17
. . .			

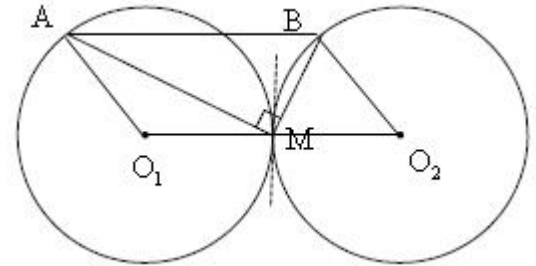
$\angle AMB = 90^\circ$.3 . M

AB

. 2 . R

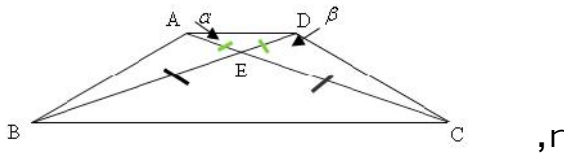
.1

. $AO_1 \parallel BO_2$ (2) $\angle O_1MO_2 = 180^\circ$ (1) . : "



	M	3	2
	O_1M	4	3
	O_2M	5	3
180° (!!!)	$\angle O_1MO_2 = 180^\circ$	6	5, 4
(1) . . .			
	$\angle BO_2M = 2r$	7	
	$O_2B = O_2M$	8	
	$\angle AMB = 90^\circ$	9	3
$\triangle BMO_2$ 180° $\triangle BMO_2$ -	$\angle BMO_2 = 90^\circ - r$	10	8, 7
	$\angle AMO_1 = r$	11	10, 9, 6
	$O_1A = O_1M$	12	
$\triangle BMO_2$ 180° $\triangle BMO_2$ -	$\angle AO_1M = 180^\circ - 2r$	13	12, 11
180° -	$AO_1 \parallel BO_2$	14	13, 7
(2) . . .			
	R	15	1
	$AO_1 = BO_2$	16	15
	AO_1O_2B	17	16, 14
	$O_1O_2 = 2R$	18	15, 6
	$AB = 2R$	19	18, 17

	R AB	20	19,9
. . .			



,r () $\angle CAD = r$.
 () ABCD
 BEC - AED

(. .) ,

$$\frac{S_{\triangle AED}}{S_{\triangle BEC}} = \left(\frac{AE}{EC}\right)^2 = \left(\frac{DE}{EB}\right)^2 :$$

$$\frac{S_{\triangle AED}}{S_{\triangle BEC}} = \left(\frac{DE}{EC}\right)^2 : , BE = CE :$$

(ADC 180°) $\angle ACD = 180^\circ - (2r + s)$
 :

$\triangle DCE$

$$\frac{DE}{\sin(180^\circ - (2r + s))} = \frac{CE}{\sin S}$$

$$\frac{DE}{CE} = \frac{\sin(2r + s)}{\sin S}$$

$$\frac{S_{\triangle AED}}{S_{\triangle BEC}} = \frac{\sin^2(2r + s)}{\sin^2 S}$$

$\frac{1}{4}$ $\sqrt{\frac{S_{\triangle AED}}{S_{\triangle BEC}}} = \frac{1}{4}$.

$$\frac{\sin(60^\circ + S)}{\sin S} = \frac{1}{4} : r = 30^\circ$$

$$\frac{\sin 60^\circ \cos S + \cos 60^\circ \sin S}{\sin S} = \frac{1}{4}$$

$$\frac{\sqrt{3}}{2} \cot S + 0.5 = 0.25 \rightarrow \frac{\sqrt{3}}{2} \cot S = -0.25$$

$$-2\sqrt{3} = \tan S$$

$$\boxed{S = 106.1^\circ}$$

$S = 106.1^\circ :$

$x \neq 3$

$$f(x) = \frac{x^2 + 6x + 12}{(x-3)^2}, \quad f(x) = \frac{x^2 + 6x + 12}{x^2 - 6x + 9} :$$

$$y = 1, \quad \lim_{x \rightarrow \infty} \frac{x^2 + 6x + 12}{x^2 - 6x + 9} = \lim_{x \rightarrow \infty} \frac{1 + \frac{6}{x} + \frac{12}{x^2}}{1 - \frac{6}{x} + \frac{9}{x^2}} = \lim_{x \rightarrow \infty} \frac{1 + 0 + 0}{1 - 0 + 0} = 1 \quad (1)$$

$$x = 3, \quad \lim_{x \rightarrow 3} \frac{x^2 + 6x + 12}{x^2 - 6x + 9} = \frac{+}{0^+} = +\infty$$

$$x = 3, \quad y = 1 :$$

$$\left(0, \frac{4}{3}\right) \quad x = 0 \quad y \quad (2)$$

$$(\Delta) \quad x$$

$$\left(0, \frac{4}{3}\right) :$$

(3)

$$f'(x) = \frac{(2x+6)(x-3)^2 - 2(x-3)(x^2+6x+12)}{(x-3)^4}$$

$$f'(x) = \frac{2(x-3)((x+3)(x-3) - (x^2+6x+12))}{(x-3)^4}$$

$$f'(x) = \frac{2(x-3)(x^2-9-x^2-6x-12)}{(x-3)^4}$$

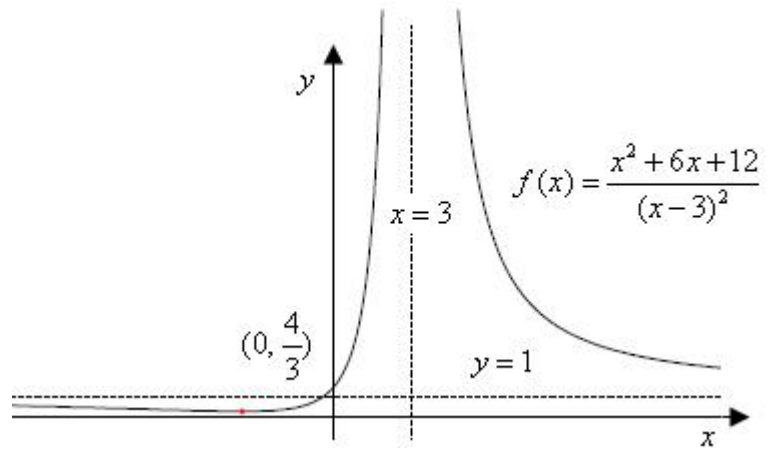
$$\boxed{f'(x) = \frac{2(x-3)(-6x-21)}{(x-3)^4}}$$

$$0 = -6x - 21 \rightarrow x = -3.5$$

$$x < -3.5 \quad x > 3 \quad -3.5 < x < 3$$

$$-3.5 < x < 3, \quad x < -3.5 \quad x > 3 :$$

(4)



$$f'(x) = \frac{2(x-3)(-6x-21)}{(x-3)^4} .$$

$$y = 0 \tag{1}$$

$$. \quad x = 3 \quad x = 3 : \quad , f'(x) = \frac{2(-6x-21)}{(x-3)^3}$$

$$x = 3 , \quad y = 0 :$$

$$. x - \quad x = -3.5 \tag{2}$$

$$. x < -3.5 \quad x > 3 \quad -3.5 < x < 3$$

$$f'(x) = \frac{2(-6x-21)}{(x-3)^3}$$

$$f''(x) = 2 \cdot \frac{-6(x-3)^3 - 3(x-3)^2(-6x-21)}{(x-3)^6}$$

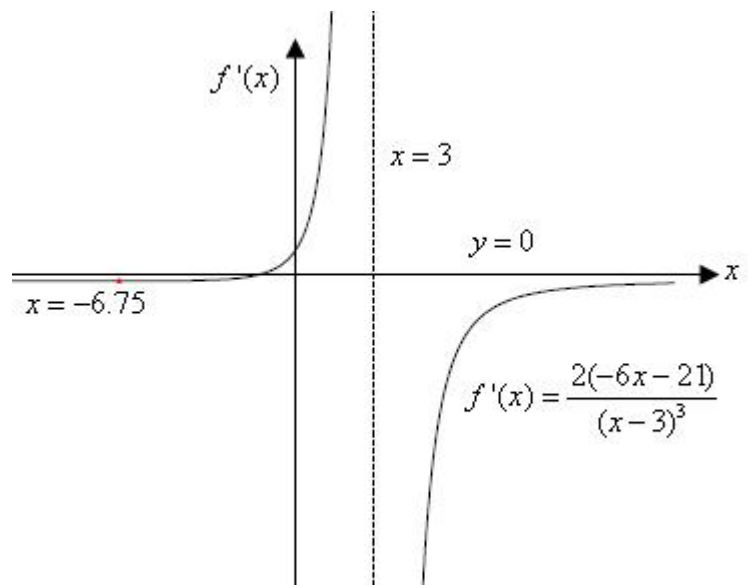
$$f''(x) = 6 \cdot \frac{-2(x-3) - (-6x-21)}{(x-3)^4}$$

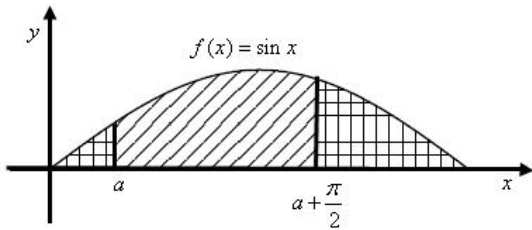
$$f''(x) = 6 \cdot \frac{-2x+6+6x+21}{(x-3)^4}$$

$$f''(x) = 6 \cdot \frac{4x+27}{(x-3)^4}$$

$$0 = 4x+27 \rightarrow x = -6.75$$

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$\frac{S_1}{S_2}$ יחס השטחים מקסימום

$$S_1 = \int_a^{a+\frac{f}{2}} (\sin x - 0) dx$$

$$S_1 = (-\cos x) \Big|_a^{a+\frac{f}{2}}$$

$$S_1 = -\cos\left(a + \frac{f}{2}\right) + \cos a$$

$$\boxed{S_1 = \sin a + \cos a} \quad \leftarrow \cos(90^\circ + r) = -\sin r$$

$$S_2 = \int_0^f (\sin x - 0) dx - S_1$$

$$S_2 = (-\cos x) \Big|_0^f - (\sin a + \cos a)$$

$$S_2 = -\cos f + \cos 0 - \sin a - \cos a$$

$$\boxed{S_2 = 2 - \sin a - \cos a}$$

$$f(a) = \frac{S_1}{S_2} = \frac{\sin a + \cos a}{2 - \sin a - \cos a}$$

$$f'(a) = \frac{(\cos a - \sin a)(2 - \sin a - \cos a) - (\sin a + \cos a)(-\cos a + \sin a)}{(2 - \sin a - \cos a)^2}$$

$$f'(a) = \frac{(\cos a - \sin a)(2 - \sin a - \cos a + \sin a + \cos a)}{(2 - \sin a - \cos a)^2}$$

$$\boxed{f'(a) = \frac{2(\cos a - \sin a)}{(2 - \sin a - \cos a)^2}}$$

$$0 = \cos a - \sin a$$

$$\tan a = 1 \quad \leftarrow \cos a, \sin a \neq 0 \quad \leftarrow 0 < a < \frac{f}{2}$$

$$a = \frac{f}{4} + f k \quad \rightarrow \quad \boxed{a = \frac{f}{4}} \quad \leftarrow k = 0$$

$$0 < a < \frac{f}{2}$$

$$f'(\frac{f}{6}) = \frac{2(\cos\frac{f}{6} - \sin\frac{f}{6})}{+} = \frac{0.73}{+} > 0, \quad f'(\frac{f}{3}) = \frac{2(\cos\frac{f}{3} - \sin\frac{f}{3})}{+} = \frac{-0.73}{+} < 0$$

$$\frac{S_1}{S_2}, \quad a = \frac{f}{4}, \quad ,$$

$$a = \frac{f}{4} :$$

$$\frac{S_1}{S_2}, \quad S_2, \quad S_1 \quad :$$

$$S_1 = \sin a + \cos a \rightarrow S_1' = \cos a - \sin a$$

$$S_1' = 0 \rightarrow \cos a = \sin a = 0 \rightarrow \tan a = 1 \rightarrow \boxed{a = \frac{f}{4}}$$

$$S_1'' = -\sin a - \cos a \rightarrow S_1'(\frac{f}{4}) < 0 \rightarrow \text{Max}$$

$$f(x) = \frac{x}{\sqrt{x^2 - 15}}$$

0 -

$$x = \pm\sqrt{15}$$

$$, x^2 - 15 > 0$$

$$x < -\sqrt{15}$$

$$x > \sqrt{15} :$$

:

:

$$f(x) = \frac{x}{\sqrt{x^2 - 15}} = \frac{x}{|x| \sqrt{1 - \frac{15}{x^2}}}$$

$$\lim_{x \rightarrow +\infty} \frac{x}{|x| \sqrt{1 - \frac{15}{x^2}}} = \lim_{x \rightarrow +\infty} \frac{x}{x \sqrt{1 - \frac{15}{x^2}}} = 1 \rightarrow \boxed{y=1} \quad \lim_{x \rightarrow -\infty} \frac{x}{|x| \sqrt{1 - \frac{15}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{x}{-x \sqrt{1 - \frac{15}{x^2}}} = -1 \rightarrow \boxed{y=-1}$$

$$\lim_{x \rightarrow \sqrt{15}^+} \frac{x}{\sqrt{x^2 - 15}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{15}}{0^+} = +\infty \rightarrow \boxed{x = \sqrt{15}} \quad \lim_{x \rightarrow -\sqrt{15}^-} \frac{x}{\sqrt{x^2 - 15}} = \lim_{x \rightarrow +\infty} \frac{-\sqrt{15}}{0^+} = -\infty \rightarrow \boxed{x = -\sqrt{15}}$$

$$x = -\sqrt{15}, x = \sqrt{15} :$$

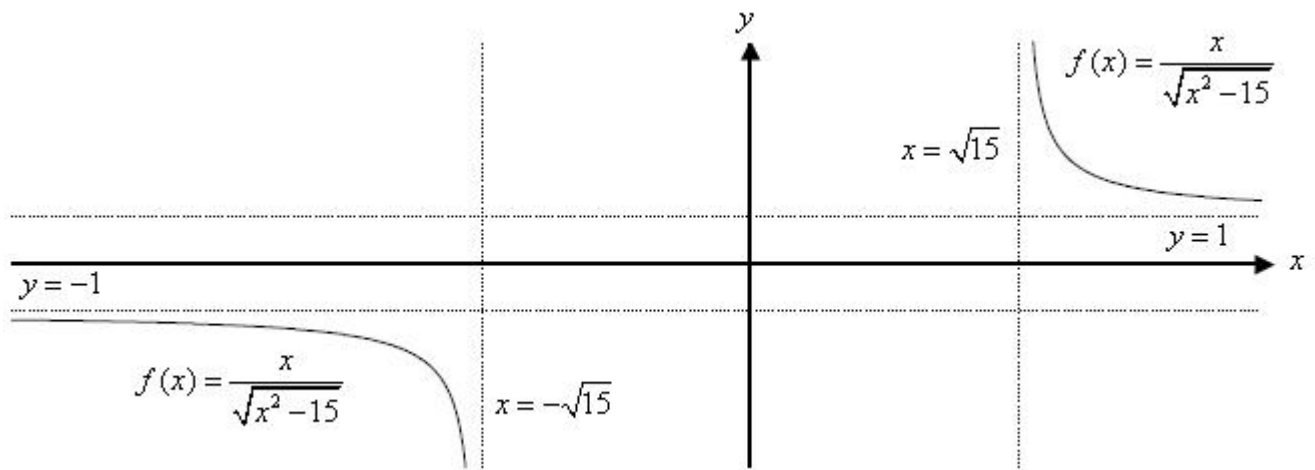
$$, y = -1, y = 1 :$$

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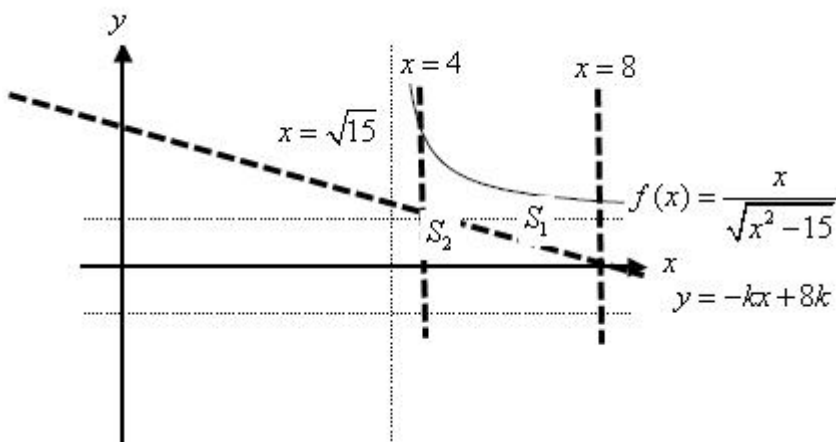
$$x < -\sqrt{15} \quad x > \sqrt{15}$$

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$$k > 0 \quad (8, 0) \quad x - \quad y = -kx + 8k$$



$$-k \cdot 4 + 8k - 0 = 4k \quad 8 - 4 = 4 : \quad S_2$$

$$S_2 = \frac{4 \cdot 4k}{2} = 8k :$$

$$, x - \quad , f(x) \quad S_1$$

$$S_2 = 8k \quad x = 8 - x = 4$$

$$x \, dx = \frac{du}{2} : \quad \frac{du}{dx} = 2x : \quad , u(x) = x^2 - 15 :$$

$$\int \frac{x}{\sqrt{x^2 - 15}} \, dx = \int \frac{1}{2\sqrt{u}} \, du = \sqrt{u} + c = \sqrt{x^2 - 15} + c$$

$$S_1 = \int_4^8 \left(\frac{x}{\sqrt{x^2 - 15}} - 0 \right) dx - 8k$$

$$S_1 = (\sqrt{x^2 - 15}) \Big|_4^8 - 8k$$

$$S_1 = (\sqrt{8^2 - 15}) - (\sqrt{4^2 - 15}) - 8k$$

$$S_1 = 6 - 8k$$

$$8k = 6 - 8k$$

$$16k = 6$$

$$\boxed{k = 0.375}$$

$$k = 0.375 :$$