

( , " ) A

( , " ) B

( " ) B A

x -  
y -  
s -

:

s - "	v - "	t -		
$\frac{3s}{4}$	x	$\frac{3s}{4x}$	A -	
$\frac{s}{4}$	y	$\frac{s}{4y}$	B -	
$\frac{s}{2}$	x	$\frac{s}{2x}$	A -	
$\frac{s}{2}$	y	$\frac{s}{2y}$	B -	

$$\frac{3s}{4x} = \frac{s}{4y} + 0.5$$

B -

$$\frac{s}{2x} + 0.5 = \frac{s}{2y}$$

A -

$$\begin{cases} \frac{3s}{4x} = \frac{s}{4y} + 0.5 \\ \frac{s}{2x} + 0.5 = \frac{s}{2y} \end{cases}$$

$$+ \begin{cases} \frac{3s}{4x} - 0.5 = \frac{s}{4y} \\ \frac{s}{2x} + 0.5 = \frac{s}{2y} \end{cases}$$

$$\frac{5s}{4x} = \frac{3s}{4y} \quad /: \frac{s}{4} > 0$$

$$\frac{5}{x} = \frac{3}{y}$$

$$\boxed{\frac{x}{y} = \frac{5}{3}}$$

.5:3

:

"

$$,A - \frac{5}{8}S$$

,5:3

,

.

$$\frac{1}{8}S$$

$$\frac{1}{8}S = b \rightarrow \boxed{S = 8b}$$

.( " ) 8b

-B

A

:

$$n=1 \quad .1 .$$

$$1^2 = 1 : \quad 1^3 = 1 :$$

$$n=1$$

,( )  $n=k$  .2

$$,(1+2+3+\dots+k)^2 = 1^3 + 2^3 + 3^3 + \dots + k^3 :$$

$$\left(\frac{k(1+k)}{2}\right)^2 = 1^3 + 2^3 + 3^3 + \dots + k^3 :$$

$$" , n = k + 1 \quad .3$$

$$\left(\frac{(k+1)(1+k+1)}{2}\right)^2 = \underbrace{1^3 + 2^3 + 3^3 + \dots + k^3}_{\downarrow} + (k+1)^3$$

$$\Leftrightarrow \frac{(k+1)^2(k+2)^2}{4} = \left(\frac{k(1+k)}{2}\right)^2 + (k+1)^3$$

$$\Leftrightarrow \frac{(k+1)^2(k+2)^2}{4} = \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$$

$$\Leftrightarrow \frac{(k+1)^2(k+2)^2}{4} = \frac{(k+1)^2(k^2 + 4(k+1))}{4}$$

$$\Leftrightarrow \frac{(k+1)^2(k+2)^2}{4} = \frac{(k+1)^2(k^2 + 4k + 4)}{4}$$

$$\Leftrightarrow \frac{(k+1)^2(k+2)^2}{4} = \frac{(k+1)^2(k+2)^2}{4}$$

,  $n = k$  ,  $n = 1$  .4

.  $n$  ,  $n = k + 1$  .

$$1^3 + 2^3 + 3^3 + \dots + n^3 = 5,833,225 .$$

$$(1+2+3+\dots+n)^2 = 5,833,225$$

$$1+2+3+\dots+n = 1,415$$

$$\frac{n(1+n)}{2} = 1,415 \rightarrow n^2 + n - 4,830 = 0$$

$$n_{1,2} = \frac{-1 \pm 139}{2} \rightarrow \boxed{n = 69} \leftarrow n > 0$$

. 69 :

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$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4} \quad 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{2}$$

- S :

-  $\bar{A}$  - A  
 -  $\bar{B}$  - B

---

N(S) = 40

$N(A) = 16 \rightarrow P(A) = \frac{N(A)}{N(S)} = \frac{16}{40} = 0.4 \rightarrow P(\bar{A}) = 0.6$

$N(B) = 12 + 16 = 28 \rightarrow P(B) = \frac{N(B)}{N(S)} = \frac{28}{40} = 0.7 \rightarrow P(\bar{B}) = 0.3$

$N(A \cap B) = 12 \rightarrow P(A \cap B) = \frac{N(A \cap B)}{N(S)} = \frac{12}{40} = 0.3$

$N(\bar{A} \cap B) = 16 \rightarrow P(\bar{A} \cap B) = \frac{N(\bar{A} \cap B)}{N(S)} = \frac{16}{40} = 0.4$

. 0.7 :

$P = 1 - 0.3^2 = 0.91$

.0.91 :

$P(A) \cdot P(B) = P(A \cap B) \quad B - A$

$P(A) \cdot P(B) = 0.4 \cdot 0.7 = 0.28 \neq P(A \cap B) = 0.3$  ,

" " " , :

$P(A \cap B) = 0.3, P(A) = 0.4 -$

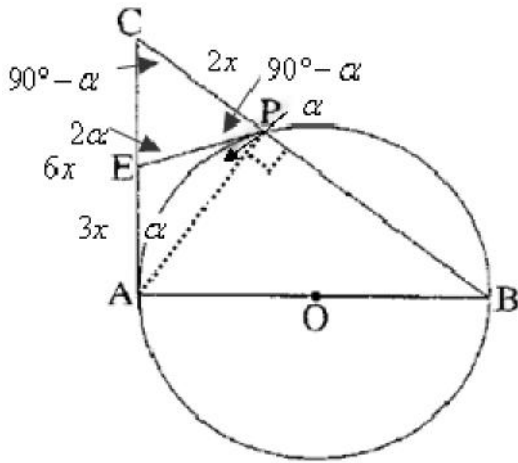
$.0.4 \cdot P(B) = 0.3 \rightarrow P(B) = 0.75 :$

$.0.75 \cdot 40 = 30 :$  , 75% -

$.30 - 12 = 18$

18 - :

"



O

AB .2  $\sphericalangle CAB = 90^\circ$  .1

.P

EP .3

$$S_{\Delta CPE} = " 2 .5 \frac{CP}{EA} = \frac{2}{3} .4 :$$

$$S_{\Delta PAB} . CE = EA . : "$$

		'	
	$\sphericalangle CAB = 90^\circ$	6	1
	O AB	7	2
	.A EA	8	7
	.P EP	9	3
(E)	EP = EA	10	9,8
	$\sphericalangle EAP = r$	11	
$\Delta EAP$	$\sphericalangle EPA = \sphericalangle EAP$	12	10
	$\sphericalangle EPA = r$	13	12,11
	$\sphericalangle APB = 90^\circ$	14	7
	$\sphericalangle CPE = 90^\circ - r$	15	14,13
$\Delta EAP -$	$\sphericalangle CEP = 2r$	16	13,11
$180^\circ \Delta CEP$	$\sphericalangle ECP = 90^\circ - r$	17	16,15
	$\sphericalangle ECP = \sphericalangle CPE$	18	17,15
$\Delta CEP$	CE = EP	19	18
	CE = EA	20	19,10
. . .			
	$S_{\Delta CPE} = " 2$	21	5
	$\frac{AC}{CE} = \frac{2}{1}$	22	20

'	$S_{\Delta CAP} = 2S_{\Delta CPE} = "$	4	23
2:1			21,22
'	$CP \cdot CB = (CA)^2$		24
			8
	$\frac{CP}{EA} = \frac{2}{3}$		25
			4
	$CP = 2x$		26
	$EA = 3x$		27
	$AC = 6x$		26,25
	$2x \cdot CB = (6x)^2$		28
	$CB = 18x$		27,20
	$PB = 16x$		29
	$\frac{PB}{CP} = \frac{16x}{2x} = \frac{8}{1}$		28,26,24
			30
			29,26
'	$S_{\Delta PAB} = 8 \cdot S_{\Delta CAP} = "$	32	31
8:1			30,26
			32
			31,23

ניתן לפתור גם באמצעות כלי טריגונומטריים

$$\Delta CAP - \sphericalangle C$$

$$\cos \sphericalangle C = \frac{1}{3} \rightarrow \sphericalangle C = 70.53^\circ$$

x

$$, \Delta CEP ,$$

$$2 = 0.5 \cdot 2x \cdot 3x \cdot \sin 70.53^\circ$$

$$x = 0.8409$$

$$AC = 6 \cdot 0.8409 = 5.0454 :$$

$$\Delta ABC -$$

$$\tan 70.53^\circ = \frac{AB}{5.0454}$$

$$AB = 14.27$$

$$\Delta ABC$$

$$\frac{14.27 \cdot 5.0454}{2} = 36$$

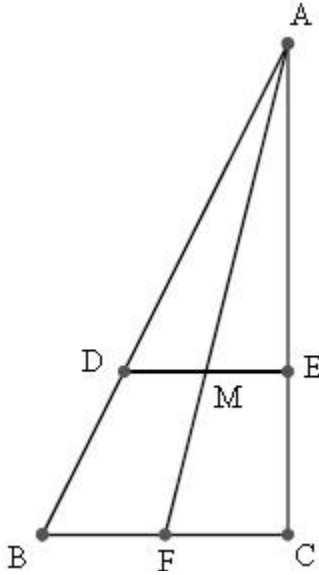
2:1

,

$$S_{\Delta CAP} = 2S_{\Delta CPE} \quad " \quad 4$$

$$S_{\Delta PAB} = \quad " \quad 32$$

"



$\angle ACB = 90^\circ$  .1  
 BC AF .2  
 $\triangle ABC$  M .3  
 DE  $\parallel$  BC .4  
 :  
 ACB DC .5  
 : "  
 $\frac{DE}{BC}$  .  
 $\triangle ABC$  - .

	BC AF	6	2
	$\triangle ABC$ M	7	3
2:1	$\frac{AM}{AF} = \frac{2}{3}$	8	7,6
	DE $\parallel$ BC	9	4
$\triangle ABF$ 1	$\frac{AD}{AB} = \frac{2}{3}$	10	9,8
$\triangle ABC$ 1	$\frac{DE}{BC} = \frac{2}{3}$	11	10,9
. . .			

( )  $\overline{ACB}$  DC

( )  $\frac{AD}{DB} = \frac{2}{1}$

(  $\triangle ABC$  )  $\frac{AC}{BC} = \frac{AD}{DB} = \frac{2}{1}$

$\tan \angle B = \frac{AC}{BC} = 2$

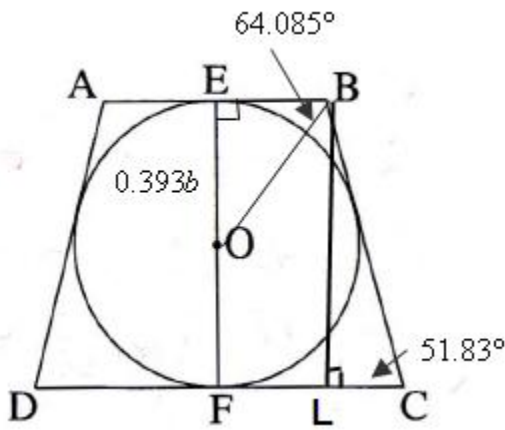
$\angle B = 63.43^\circ$

(  $180^\circ - \triangle ABC$  )  $\angle A = 26.57^\circ$

$\angle A = 26.57^\circ$  ,  $\angle B = 63.43^\circ$   $\triangle ABC$  - :

"





$$\begin{aligned} & \angle C \\ & \angle (\sin \angle C)^2 = \sin(90^\circ - \angle C) \\ & 1 - (\cos \angle C)^2 = \cos \angle C \\ & (\cos \angle C)^2 + \cos \angle C - 1 = 0 \rightarrow (\cos \angle C)_{1,2} = \frac{-1 \pm \sqrt{5}}{2} \\ & (\cos \angle C)_1 = 0.618 \rightarrow \boxed{\angle C = 51.83^\circ} \\ & (\cos \angle C)_2 < -1 \rightarrow \emptyset \leftarrow 0 < \cos \angle C < 1 \end{aligned}$$

( ) BL ⊥ CD  
 ( ) O EF  
 ( ) ∠BEF = ∠EFC = 90°  
 ( ) BEFL  
 ( ) BL = 2r

ΔBCL

$$\begin{aligned} \frac{BL}{BC} &= \sin \angle C \Rightarrow \frac{2r}{b} = \sin 51.83^\circ \\ r &= \frac{b \cdot \sin 51.83^\circ}{2} = 0.393b \end{aligned}$$

0.393b :  
 . AB  
 (180° ) ∠ABC = 128.17°

( ) ∠EBO =  $\frac{128.17^\circ}{2} = 64.085^\circ$

ΔEBO

$$\begin{aligned} \tan \angle EBO &= \frac{r}{EB} \\ \tan 64.085^\circ &= \frac{0.393b}{EB} \\ EB &= \frac{0.393b}{\tan 64.085^\circ} \\ EB &= 0.191b \end{aligned}$$

. AB = 2BE - OE ΔAOB -  
 $\boxed{AB = 0.382b}$

. AB = 0.382b :

$$f(x) = \frac{1}{\cos x}$$

$$f(-x) = f(x)$$

$$\cos x = \cos(-x) \rightarrow f(-x) = \frac{1}{\cos(-x)} = \frac{1}{\cos x} = f(x)$$

$$0 \leq x \leq 2f$$

$$x \neq \frac{f}{2} + fk \tag{1}$$

$$\lim_{x \rightarrow \pm \frac{f}{2}} f(x) = \pm \infty \rightarrow x = \frac{f}{2}, x = \frac{3f}{2}$$

$$x = \frac{f}{2}, x = \frac{3f}{2}, 0 \leq x \leq 2f, x \neq \frac{f}{2}, \frac{3f}{2} \tag{2}$$

$$f(0) = \frac{1}{\cos 0} = 1 \rightarrow (0, 1), f(2f) = \frac{1}{\cos 2f} = 1 \rightarrow (2f, 1)$$

$$f'(x) = \frac{\sin x}{\cos^2 x}$$

$$\sin x = 0 \rightarrow x = fk$$

$$f(f) = \frac{1}{\cos f} = -1 \rightarrow (f, -1) \quad x=f \quad k=1$$

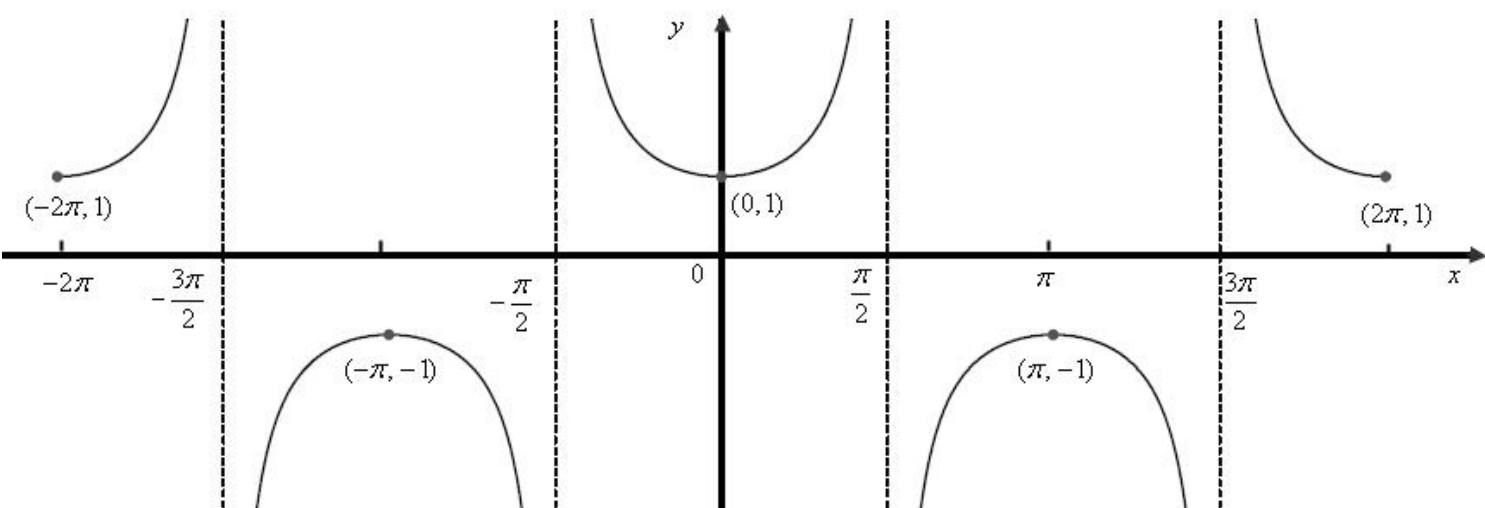
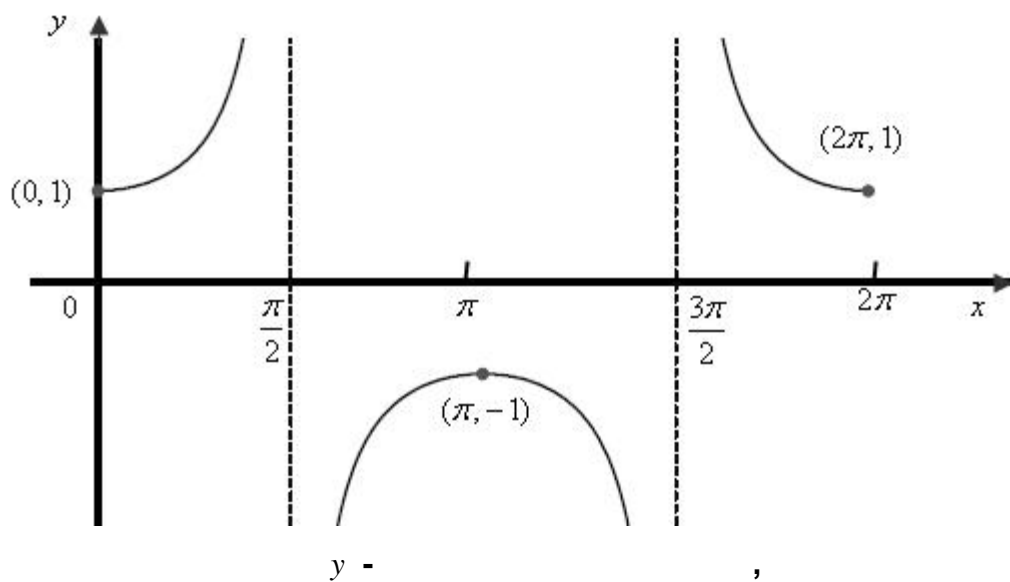
$$f'(\frac{f}{6}) = \frac{0.5}{+} > 0, f'(\frac{2f}{3}) = \frac{0.866}{+} > 0, f'(\frac{4f}{3}) = \frac{-0.866}{+} > 0, f'(\frac{11f}{6}) = \frac{-0.5}{+} < 0$$

0	$\frac{f}{6}$	$\frac{f}{2}$	$\frac{2f}{3}$	$f$	$\frac{4f}{3}$	$\frac{3f}{2}$	$\frac{11f}{6}$	$2f$	$x$
	+		+	0	-		-		$f'(x)$
<b>min</b>	$\nearrow$		$\nearrow$	<b>max</b>	$\searrow$		$\searrow$	<b>Min</b>	

$$(2f, 1), (f, -1), (0, 1)$$

$$0 \leq x \leq 2\pi$$

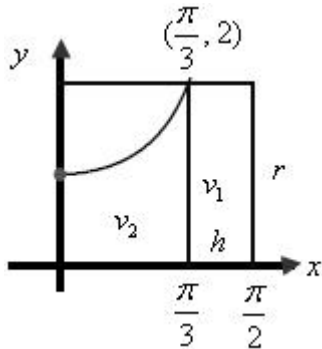
(3)



$(\frac{f}{3}, 2)$

$k=0 \quad \frac{1}{\cos x} = 2 \rightarrow \cos x = 0.5 \rightarrow x = \pm \frac{f}{3} + 2fk$

$\frac{f}{2} - \frac{f}{3} = \frac{f}{6}$



$v_1 = f r^2 h = f \cdot 2^2 \cdot \frac{f}{6} = \frac{2}{3} f^2$

$v_2 = f \int_0^{\frac{f}{3}} (\frac{1}{\cos x})^2 dx$

$v_2 = f \int_0^{\frac{f}{3}} \frac{1}{\cos^2 x} dx$

$v_2 = f \tan x \Big|_0^{\frac{f}{3}}$

$v_2 = f \tan \frac{f}{3} - f \tan 0$

$v_2 = f \sqrt{3}$

$\frac{2}{3} f^2 + f \sqrt{3} = 12.02$

$-\infty < x < \infty$

$2fk$

$y = \cos x$

$f(x) = \frac{1}{\cos x}$

$(2fk, 1)$  (1)

$(f + 2fk, -1)$  (2)

$f'(x) = \frac{\sin x}{\cos^2 x} \rightarrow f''(x) = \frac{\cos^3 x + 2 \cos x \sin^2 x}{\cos^4 x} \rightarrow f''(x) = \frac{\cos x (\cos^2 x + 2 \sin^2 x)}{\cos^4 x}$

$\cos x = \pm 1$

$\sin x = 0$

$x = fk$

$\cos x$

$\cos x = 1 > 0 \quad x = 2fk$  (1)

$(2fk, 1)$  :

$\cos x = -1 < 0 \quad x = f + 2fk$  (2)

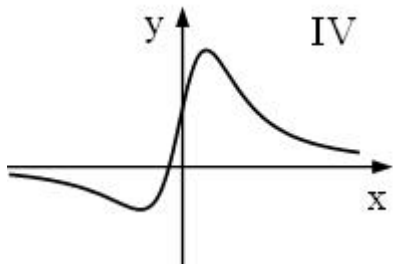
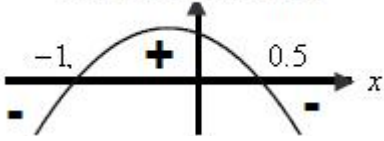
$(f + 2fk, -1)$  :

$$f''(x) = \frac{-6x^2 - 3x + 3}{\sqrt{(1+x^2)^5}}$$

$f'(x)$

$f(x)$

סימן הנגזרת השנייה



$$-6x^2 - 3x + 3 = 0$$

$$x_{1,2} = \frac{3 \pm 9}{-12} \rightarrow x = -1, x = 0.5$$

$$-6x^2 - 3x + 3$$

$$-1 < x < 0.5 \quad f''(x) > 0$$

$$x < -1 \quad x > 0.5 \quad f''(x) < 0$$

IV :

$f''(x)$

(1)

$$x < -1 \quad x > 0.5 \cap$$

$$-1 < x < 0.5 \cup$$

:

$$f'(x) \quad 0 \quad -1 \quad IV$$

(2)

$$f(x) \quad ( \quad )$$

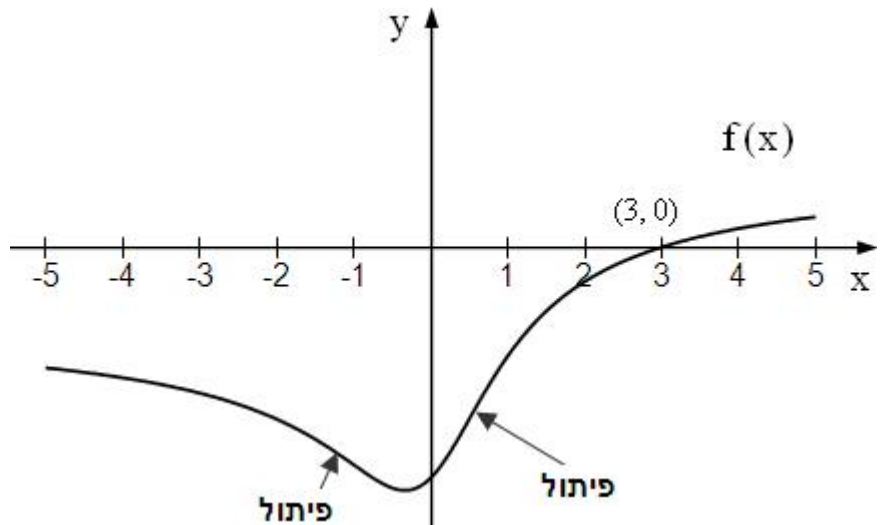
$$0 \quad -1 \quad :$$

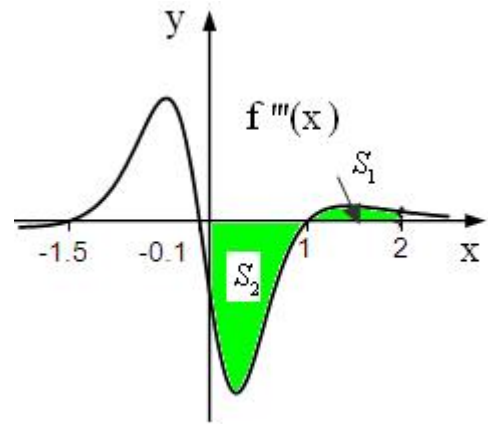
$f(x)$

(2) - (1)

(3, 0)

(3)





$$S_1 = \int_1^2 (f'''(x) - 0) dx = \left[ f''(x) \right]_1^2 = f''(2) - f''(1) = \frac{-6 \cdot 2^2 - 3 \cdot 2 + 3}{\sqrt{(1+2^2)^5}} - \frac{-6 \cdot 1^2 - 3 \cdot 1 + 3}{\sqrt{(1+1^2)^5}} = -0.483 - (-1.061) = 0.578$$

$$S_2 = \int_0^1 (0 - f'''(x)) dx = - \left[ f''(x) \right]_0^1 = -f''(1) + f''(0) = 1.061 + 3 = 4.061$$

$$S = S_1 + S_2 = 0.578 + 4.061 = 4.639$$

. " 4.639 :

35806

11

, (1, 0)

,  $g(x) = x^2 - x$

A

, O -

,  $f(x) = -a^2x^2$

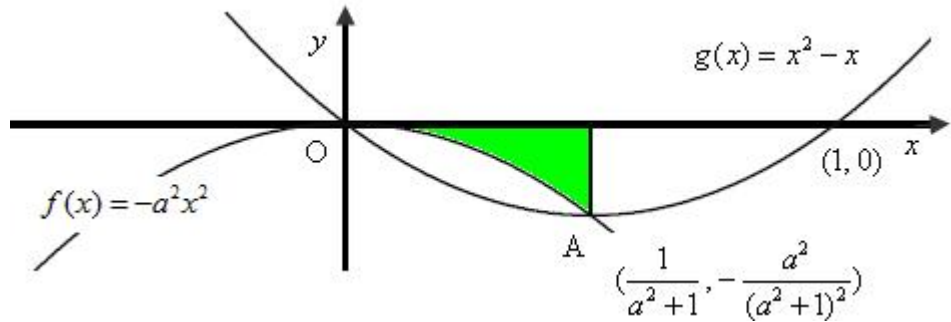
$$-a^2x^2 = x^2 - x \quad /: x_A > 0$$

$$-a^2x = x - 1 \rightarrow 1 = (a^2 + 1)x$$

$$x = \frac{1}{a^2 + 1} \rightarrow y = -\frac{a^2}{(a^2 + 1)^2} \rightarrow \boxed{A\left(\frac{1}{a^2 + 1}, -\frac{a^2}{(a^2 + 1)^2}\right)}$$

$$A\left(\frac{1}{a^2 + 1}, -\frac{a^2}{(a^2 + 1)^2}\right) :$$

השטח הירוק מתחת ל-  $g(x)$  ופנימה ל-  $f(x)$ .



$$S = \int_0^{\frac{1}{a^2+1}} (0 + a^2x^2) dx$$

$$S = \left[ \frac{a^2x^3}{3} \right]_0^{\frac{1}{a^2+1}}$$

$$S = \frac{1}{3} \left( a^2 \left( \frac{1}{a^2+1} \right)^3 - a^2 \cdot 0^3 \right)$$

$$\boxed{S = \frac{1}{3} \cdot \frac{a^2}{(a^2+1)^3}}$$

.  $0 < x < 1$

A

x -

,

$$S = \frac{1}{3} \cdot \frac{a^2}{(a^2 + 1)^3}$$

$$S' = \frac{1}{3} \cdot \frac{2a(a^2 + 1)^3 - 3(a^2 + 1)^2 \cdot 2a \cdot a^2}{(a^2 + 1)^6}$$

$$S' = \frac{1}{3} \cdot \frac{2a(a^2 + 1 - 3a^2)}{(a^2 + 1)^4}$$

$$S' = \frac{1}{3} \cdot \frac{2a(1 - 2a^2)}{(a^2 + 1)^4}$$

$$0 = 1 - 2a^2$$

$$a = \pm\sqrt{0.5}$$

$$S'(-1) = \frac{1}{3} \cdot \frac{2 \cdot (-1) \cdot (1 - 2 \cdot (-1)^2)}{+} = \frac{1}{3} \cdot \frac{-}{+} < 0, \quad S'(-0.5) = \frac{1}{3} \cdot \frac{2 \cdot (-0.5) \cdot (1 - 2 \cdot (-0.5)^2)}{+} = \frac{1}{3} \cdot \frac{-}{+} < 0 \rightarrow \text{Max}$$

$$S'(0.5) = \frac{1}{3} \cdot \frac{2 \cdot 0.5 \cdot (1 - 2 \cdot 0.5^2)}{+} = \frac{1}{3} \cdot \frac{+}{+} > 0, \quad S'(1) = \frac{1}{3} \cdot \frac{2 \cdot 1 \cdot (1 - 2 \cdot 1^2)}{+} = \frac{1}{3} \cdot \frac{-}{+} < 0 \rightarrow \text{Max}$$

$$x_A = \frac{1}{1+0.5} = \frac{2}{3} \rightarrow y_A = \frac{-0.5}{(1+0.5)^2} = -\frac{2}{9} \rightarrow \boxed{A\left(\frac{2}{3}, -\frac{2}{9}\right)}$$

,

,

$$a = -\sqrt{0.5}$$

$$a = +\sqrt{0.5}$$

.

$$\cdot A\left(\frac{2}{3}, -\frac{2}{9}\right)$$

.

$$, A\left(\frac{2}{3}, -\frac{2}{9}\right)$$

: